DIFFRACTION EFFECTS IN THE SCATTERING OF NEUTRONS, μ MESONS AND ELECTRONS BY NUCLEI

PART II

2 THE SCATTERING OF FAST μ MESONS AND ELECTRONS BY NUCLEI

2.1-*Experimental Evidence on the Specific Interaction of μ Mesons with Nucleons*

The first informations about the specific interaction of μ mesons with nucleons were obtained by different authors (Blackett and Wilson, 1938; Vargus, 1939; Wilson, 1940; Code, 1941, Shutt, 1942, 1946; Sinha, 1945; Scott and Snyder, 1948) who measured the anomalous scattering of μ mesons by nuclei, *i.e.*, the scattering observed in addition to that to be expected from the purely electric forces. Most of these experiments that were performed with μ mesons of kinetic energies of the order of 300 MeV, gave an upper limit of the cross section of about 5×10^{-2} cm.^{2}/nucleon, *i.e.*, a value appreciably smaller than that expected under the assumption that the mesons observed in cosmic rays at sea-level can be identified with the particles responsible of the nuclear forces.

All the measurements in questions were based on very poor statistics and generally open to various criticisms that induced one to suspect that the scattering cross-section could be still smaller than the value quoted above.

A much lower upper limit can be deduced by the consideration of the experimental results of Conversi, Pancini and Piccioni (1945, 1946, 1947), on the behaviour of μ mesons at the end of the range in materials of different atomic number. According to the discussions first given by Fermi, Teller and Weisskopf (1947) and detailed, later on, by other authors (Ferretti, 1948; Frölich and Wheeler, 1949), the above-mentioned effect shows that the interaction constant of low energy μ mesons with nucleons, is 10^6 times smaller than that expected under the assumption that μ mesons are the particles responsible of the nuclear forces.

In order to deduce from these data an upper limit of the scattering cross section we need to introduce some assumption about the mechanism of the processes of absorption and scattering of μ mesons by nucleons. If, for instance, we assume that the absorption can be interpreted in terms of a charge-exchange reaction, as discussed by Tiomno and Wheeler (1949), and that the neutral particle emitted in such a process is comparable with the absorbed μ meson, the scattering cross section turns out to be 10^{12} times smaller than the upper limit set by the above-mentioned scattering experiments. One arrives at similar conclusions also from other resonable assumption about the mechanism of these processes (Wheeler, 1949; Tiomno and Wheeler, 1949).

It is to be noted, however, that these considerations contain the implicit assumption that conclusions drawn from experimental observations on the behaviour of μ mesons almost at rest, can be extrapolated to the case of μ mesons of some hundred MeV kinetic energy.

Therefore we thought worthwhile to carry on an experiment having the character of direct observation of large angle scattering of fast μ mesons by nucleons.

The experimental set-up (Amaldi and Fidecaro, 1950, 1951) is shown schematically in figure 2.1.

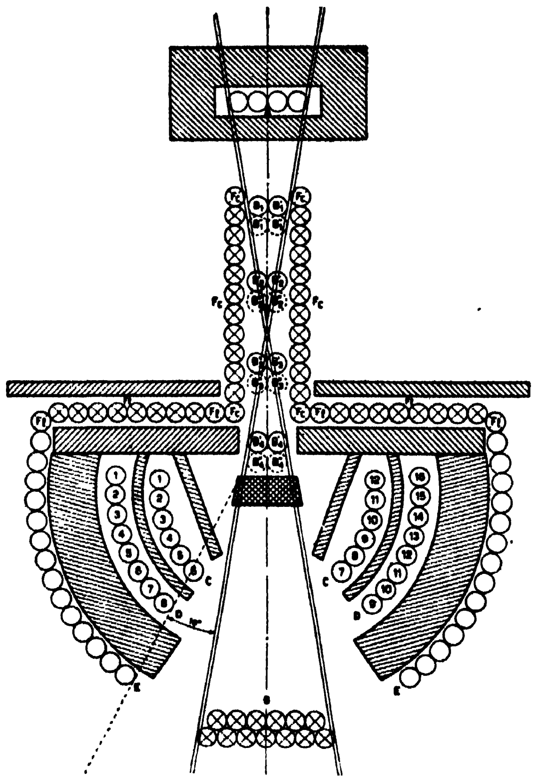

FIG. 2.1

While we refer to the original papers for details of the experimental set-up and of the discussion of the results of the measurements, we will recall the following main conclusions: in about 1,500 hours we recorded about 0.5 10^6 μ mesons crossing the scatterer, and only 4 scattered particles in the low energy band and 3 particles in the high energy band.

Most of these particles are probably protons whose percentage in the cosmic radiation at sea-level is still rather uncertain. Making a rather conservative evaluation of the contribution given by protons to the observed scattered particles, we could establish the following upper limits of the anomalous scattering cross-sections* :

for $200 \lesssim T_\mu \lesssim 320$ MeV $\quad \sigma \lesssim 4.5 \times 10^{-29}$ cm.^{2}/nucleon ... (2.1) (2.1)

for $320 \lesssim T_\mu$ MeV $\quad \sigma \lesssim 2.3 \times 10^{-30}$ cm.^{2}/nucleon ... (2.2)

In the high energy band (corresponding to an average kinetic energy $\bar{T}_\mu = 900$ MeV) our upper limit is about 200 times smaller than that deduced by the preceding scattering experiments but still enormously larger than that deduced from the Conversi Pancini and Piccioni effect.

With the same experimental set-up we could establish an upper limit of about

$$10^{-30} \text{ cm.}^2/\text{nucleon} \qquad \ldots \quad (2.3)$$

for the cross section for production of penetrating showers by μ mesons at sea-level : this result refers to penetrating showers containing at last one particle emitted at an angle larger than 18°.

Two remarks about these values can be added. The first one is that although our upper limits are not extremely small, they are sufficiently small and refer to the right energy interval for the considerations that we are going to develop at a later time.

* In order to visualize the specific interaction of μ mesons and nucleons some authors (Wheeler, 1949; Tiomno and Wheeler, 1949; Fermi and Marshall, 1947, Havens, Rabi, Rainwater, 1947) represent the supplementary interaction with a potential well extending over a region that is small if compared with the meson's wavelength.

From the perturbation theory, one gets the following order of magnitude of the cross section

$$\sigma \lesssim \frac{2\pi}{\hbar} W^2 \frac{4\pi p^2}{c^2 h^3} \qquad \ldots \quad (\mathrm{I})$$

where W is the depth of the potential well multiplied by its volume. From equation (I) one gets

$$W \lesssim \frac{c^2 h^2}{2^{1/2}(2\pi)^{3/2}} \frac{\sigma^{1/2}}{cp} \sim 3 \times 10^{27} \frac{\sigma^{1/2}(\mathrm{cm}^2)}{(cp)_{\mathrm{ev}}} \times \frac{4\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^3 \qquad \ldots \quad (\mathrm{II})$$

where the classical radius of the electron has been introduced only to allow an easy evaluation of the orders of magnitude. Introducting in equation (II) $\sigma \sim 2.3 \times 10^{-30}$ and $cp \sim 10^9$eV, we get

$$W \lesssim 5{,}000 \text{ eV} \times \frac{4\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^3$$

which is convenient for a comparison with the results obtained for the interaction among slow neutrons and electrons (Fermi and Marshall, 1947, Havens, Rabi and Rainwater, 1947).

We note that in the present case of the interaction among fast μ mesons and nucleons, the wavelength λ^* of the incident particle is comparable with the classical radius of the electron and therefore the preceding considerations make sense only if the dimensions of the potential well were smaller than e^2/mc^2 and its depth correspondingly larger.

undergo a transition from the ground state to an excited state. We note that we have to expect that the results obtained for the "one particle model" are better in the case of the coherent scattering, in which only the wave function of the ground state of the nucleus is involved, than in the case of the incoherent scattering (Weisskopf, 1950).

Most of the following considerations and conclusions are independent of the particular shape chosen for the potential well. Considering, however, that in order to be able to carry on the calculations in a simple form up to numerical results, we need, at a later time, to specify the potential well in a convenient way, we will choose from now the "parabolic well". With such an assumption, which gives satisfactory results only for light nuclei (Heisenberg, 1935 ; Bethe and Bacher, 1936), each nucleon will be represented by a three dimensional isotropic harmonic oscillator, whose energy interval $W=h\nu$ is the only parameter that we have to adjust in such a way that our model reproduces correctly one conveniently chosen experimental feature of the nucleus. Considering the type of phenomena that we are investigating, we thought more convenient to adjust the length

$$a=\left(\frac{\hbar^2}{M_p w}\right)^{1/2} \qquad \ldots \quad (2.4)$$

in such a way that our model has the experimental dimensions of the nucleus instead of the experimental binding energy (Wheeler, 1949 ; Tiomno and Wheeler, 1949), namely we have applied the Pauli principle to the nucleons, and we have imposed that the mean value of the square of the distance of the last nucleon is equal to the square of

$$R_0=\tfrac{1}{2}\frac{e^2}{m_e c^2}A^{1/3}.$$

So we get

$$a=\frac{e^2}{m_e c^2}\,\frac{A^{1/3}}{\sqrt{2(2m+1)}}$$

where m is the quantum number of the last occupied level.

Going back to our scattering problem we note that if we neglect the spin of the particles, the electromagnetic interaction between the μ meson and the nuclear protons reduces to the Coulomb interaction. Following an elementary analytical procedure similar to that given by Bethe (1930) in the discussion of the collision of electrons of a few keV against atoms, one gets the following equation for the total cross section

$$\frac{d\sigma}{d\omega}=R\left(F^2+\frac{\pi}{Z}\right) \qquad \ldots \quad (2.5)$$

where

$$R=\tfrac{1}{4}\left(\frac{e^2}{m_e c^2}\right)^2\left[\frac{m_e c^2 E_0}{(cp_0)^2}\right]^2\frac{Z^2}{\sin^4\frac{\vartheta}{2}} \qquad \ldots \quad (2.6)$$

is the Rutherford cross section for particles of total energy E_0 and momentum p_0, colliding against a point charge Ze.

The only remark that we like to make about the deduction of (2.5), is that, if we take, for sake of simplicity, one of the axis of the frame of reference, say the X_1-axis, in the direction of the vector

$$\vec{k}=\vec{K}-\vec{K}_0 \qquad \dots \quad (2.7)$$

($\vec{K}_0$ and $\vec{K}$ are the wave vectors of the incident and of the scattered meson) the selection rules concerning our problem can be expressed in the following simple form

$$m_2=n_2,\ m_3=n_3 \qquad \dots \quad (2.8)$$

i.e. to the scattering of the incident particle contributes only the oscillation of the protons in the direction of the vector $\vec{k}$.

In (2.5) the term RF^2 represents the coherent scattering ($m_1=n_1$). F is nuclear form factor that can be put in the form

$$F=\frac{1}{Z}\{z_1f_1+z_2f_2+z_3f_3+\dots\} \qquad \dots \quad (2.9)$$

where z_i is the number of protons in the level i and the f_i are the following functions of $x=\frac{Q_0^2}{2}$

$$f_1=e^{-x/2};\ f_2=e^{-x/2}(1-\tfrac{1}{3}x)\ ;\ f_3=e^{-x/2}(1-\tfrac{2}{3}x+\tfrac{1}{12}x^2) \qquad \dots \quad (2.10)$$
$$f_4=e^{-x/2}(1-x+\tfrac{1}{4}x^2-\tfrac{1}{60}x^3);\dots$$

$$Q_0=ak_0=\frac{2a}{\lambda_0}\sin\frac{\vartheta}{2}. \qquad \dots \quad (2.11)$$

If we neglect terms in x^2, x^3, ..our F is identical with the F of Williams provided the length b representing, according to this author, the dimensions (not better defined) of the nucleus, is related to our a by the equation

$$b=a\sqrt{2}\left[\tfrac{1}{2}+\frac{z_2+2z_3+3z_4}{3Z}\right] \qquad \dots \quad (2.12)$$

i.e. b must be, for light elements, about 30% smaller than the corresponding R_0.

The second term in (2.4), *i.e.*, $R\pi/Z$, corresponds to the incoherent scattering, namely the scattering accompanied by excitation of any one of the nuclear oscillations in the direction of $\vec{k}$, from its initial state n_1 to any free final state m_1,

$$\pi=\frac{1}{Z}\Sigma\alpha_{m_1-n_1}e^{-x}P_{m_1n_1}(x) \qquad \dots \quad (2.13)$$

where the sum has to be extended over all the free final states m_1 and over all the occupied initial states n_1 and

$$\alpha_{m_1-n_1}=16\sin^4\frac{\vartheta}{2}\ \frac{(1-y)\phi(y)}{\{1+\phi^2(y)-2\phi(y)\cos\vartheta\}^{1/2}} \qquad \dots \quad (2.14)$$

$$\phi(y)=\left\{1-\frac{1}{\beta_0^2}\,y(2-y)\right\}^{1/2} \qquad y=(m_1-n_1)\frac{W}{E_0}. \tag{2.15}$$

P_{nm} are polinomials of the following type

$$P_{0m}=\frac{1}{m!}x^m\ ;\ P_{1m}=\frac{1}{m!}\{x^{m+1}-2mx^m+m^2x^{m-1}\}\ ; \tag{2.16}$$

$$x=\frac{Q^2}{2}\ ;\quad Q=ak\quad \frac{ap_0}{\hbar}\{1+\phi^2(y)-2\phi(y)\cos\vartheta\}^{1/2} \tag{2.17}$$

In figure 2.2, we give, as an example, the quantity F^2 (marked C), the quantity π/Z (marked I), their sum (marked T) and the F^2 according to Willams, taking, as it is usually done, $b=R_0$, for μ mesons of $E_0=200$ and $E_0=600$ MeV, colliding against C.

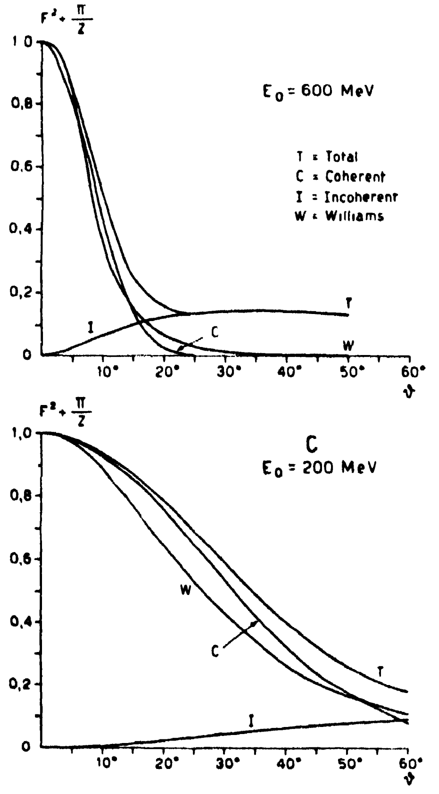

FIG. 2.2

The scattering of particles of spin 1/2 can be calculated following a procedure quite similar to that of Moller (1932), in which the spin-spin interaction and the retardation due to the finite value of the velocity of propagation of the electromagnetic field, are taken into account.

While for the incident and scattered meson we have used the Dirac wave functions of a free particle, for the nuclear protons we used the Darwin approximation according to which terms of the order of the square of

$$\frac{1}{K}=\frac{\hbar}{M_pc}$$

are neglected. Such an approximation, which is better satisfied in our case of nuclear protons than in the case of the atomic electrons allows one to specify the potential well as a parabolic one (*i.e.* a well of infinite depth which would give rise to complications if treated exactly in the Dirac theory).

A second consequence of such an approximation is that the Pauli additional term that can be introduced in the Lagrange function in order to take into account the anomalous magnetic moment of the protons $\left(\mu_p=\gamma\frac{e\hbar}{M_p c}=\gamma\frac{e}{K}\right)$ can be neglected. In fact the corresponding term appearing in the current equation (the so called polarisation current) gives rise in the expression of the cross section only to terms of the order of $\frac{1}{K^2}(\gamma-1)^2$ as one can see from a direct calculation as well from the results of Corben and Schwinger (1940) on the collision between two free particles.

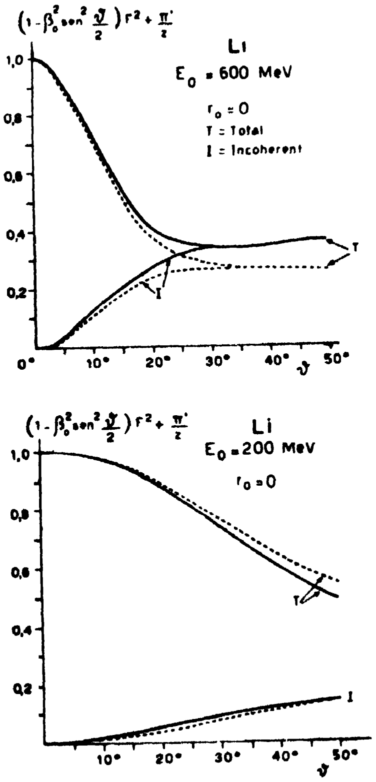

FIG. 2.3

By carrying on the calculations, one gets, the following in the first Born approximation (Amaldi, Fidecaro and Mariani, 1950) instead of (2.5)

$$\frac{d\sigma}{d\omega}=R\left\{\left(1-\beta_0^2\sin^2\frac{\vartheta}{2}\right)F^2+\frac{\pi'}{Z}\right\} \tag{2.18}$$

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where F is still given by (2.9), while

$$\pi' = \frac{1}{Z} \Sigma\, \alpha'_{m_1-n_1} e^{-u} P_{n_1 m_1}(x)$$

$$\alpha'_{m_1-n_1} = \frac{4\phi(y)\sin^4\frac{\vartheta}{2}}{\left\{1+\phi^2(y)-2\phi(y)\cos\vartheta-\left(\frac{y}{\beta_0}\right)^2\right\}^2} \left\{(2-y)^2-\beta_0^2(1+\phi^2(y)-2\phi(y)\cos\vartheta)\right.$$

$$-\frac{4\beta_0 p_0}{\hbar K} \frac{1-\phi^2(y)-y[1-\phi(y)\cos\vartheta]}{\{1+\phi^2(y)-2\phi(y)\cos\vartheta\}^{1/2}} \quad 1+\phi^2(y)-2\phi(y)\cos\vartheta$$

$$p_0 a \quad m_1 \quad \frac{I_{n_1 m_1 - 1}|}{I_{n_1 m_1}|} + \frac{1}{2}\frac{|I_{n_1 m_1+1}|}{|I_{n_1 m_1}|}$$

$$I_{n_1 m_1} = a \int^{+\infty} \phi_{n_1}(\xi)\phi_{m_1}(\xi) e^{iQ\xi} d\xi$$

ϕ_{n_1}, ϕ_{m_1} are the wave functions of a harmonic oscillator in state n_1, m_1,... Figure. 2.3 shows a comparison between equation 2.10 (thickly drawn line) and 2.5 (broken line) for μ mesons of $E_0 = 100$ MeV, colliding against Li nuclei.

2.3—*The Electromagnetic Dimensions of the Nucleons*

In the preceding discussion we have tacitly assumed that each proton acts on the μ meson as a point-charge. Now it is evident that such an assumption is not justified: on account of the nuclear forces the proton has radius of about $1.4 \cdot 10^{-13}$ cm, but as long as we know, there is no definite experimental evidence in favour of or against the assumption that the electric charge (and eventually the magnectic moment) of the proton is spread on the same spatial dimensions covered by the nuclear forces. The problem is obviously connected with the nature of the nuclear forces and the existence of processes of emission and absorption of mesons (π, τ,...) by a proton.

By the way it is to be noted that on account of these processes of emission and absorption of mesons by nucleons, one has to expect an electromagnetic interaction of μ mesons also with neutrons similar to that of electrons with neutrons, in addition to the interaction due to the spins.

All these questions, however, are of a quite different type from those we have proposed to consider in the present discussion and have been mentioned only in order to show the "a priori" possibility that the electromagnetic radius of a proton is finite and different from its radius as determined by the nuclear forces.

It is also obvious that similar arguments can be applied to any other particle different from the nucleons, in particular to the μ mesons.

In the present discussion we shall limit ourselves to discuss, in a pure phenomenological way, the influence of a finite electromagnetic radius of the proton on the Coulomb scattering of μ mesons by light nuclei (Amaldi, Fidecaro and Mariani, 1950).

As a first approach to this problem we have assumed that the charge e of the proton is distributed around its center according to a Gaussian law.

$$\rho(r) = e \frac{1}{\pi^{3/2} r_0^3} e^{-\left(\frac{r}{r_0}\right)^2} \qquad \ldots \quad (2.19)$$

It is then immediately shown that the matrix element corresponding to a transition $n \rightarrow m$ of the nucleus, induced by the incident point-charge μ meson, calculated for a gaussian proton $H^{(g)}_{nm}$ is connected to the corresponding matrix element H_{nm}, calculated for point-charge protons, by the simple relation,

$$H^{(g)}_{nm} = T H_{nm} \qquad \ldots \quad (2.20)$$

with

$$T = e^{-q^2/4} \; ; \; q = k r_0 = r_0 \frac{|\vec{p} - \vec{p}_0|}{\hbar} \qquad \ldots \quad (2.1)$$

This result introduced in the expression of the cross section, allows one immediately to deduce the influence of a finite radius of the charge distribution of the proton on the Coulombian scattering of fast μ meson.

In Figure 2.4 the quantity $F^2 + \pi/Z$ is plotted as a function of the scattering angle for different values of r_0 and for μ meson of $E_0 = 200$ and 600 MeV, colliding against C nuclei.

The used numerical values of r_0 correspond to the Compton wave length of respectively nucleons and π mesons, which can be considered as a lower and an upper limit of the electromagnetic dimensions of the nucleons.

Figure 2.5 is a plot of the spectra of the inelastically scattered mesons by C for two values of r_0 and $E_0 = 600$ MeV ; from these figures we see that the electromagnetic scattering of μ mesons by light nuclei is very sensitive to the electromagnetic dimensions of the nucleons.

2.4—*Discussion and Generalization of the Preceding Conclusions*

The electromagnetic scattering has been calculated in sections 2.2 and 2.3 using very rough models for both the nucleus and the nucleons. Therefore, it seems of some interest to discuss how far do our numerical results depend on the particular models employed.

(*a*) *The model of the nucleus.* First we have changed the rule of filling the nuclear shells. In the calculations reported above the nucleons have been distributed uniformly in all the degenerate substates belonging to an unfilled shell.

We tried to put the nucleons in the degenerate substates according to the symmetries suggested by the one particle model developed mainly by Mayer (1948, 1949) and Haxel, Jensen and Suess (1948, 1950). No difference at all

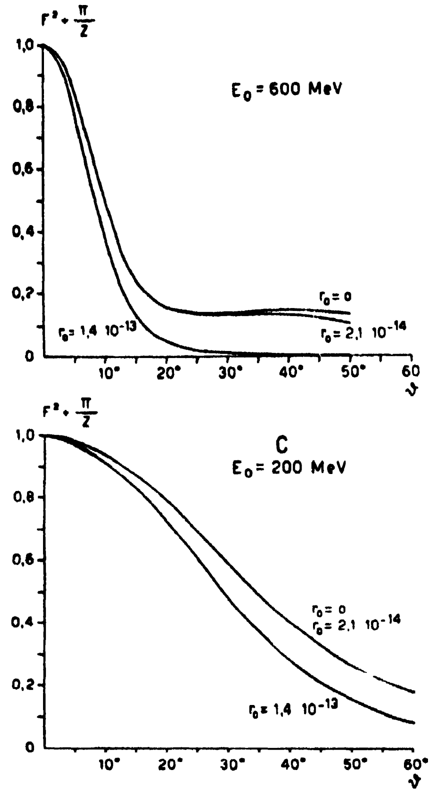

FIG. 2.4

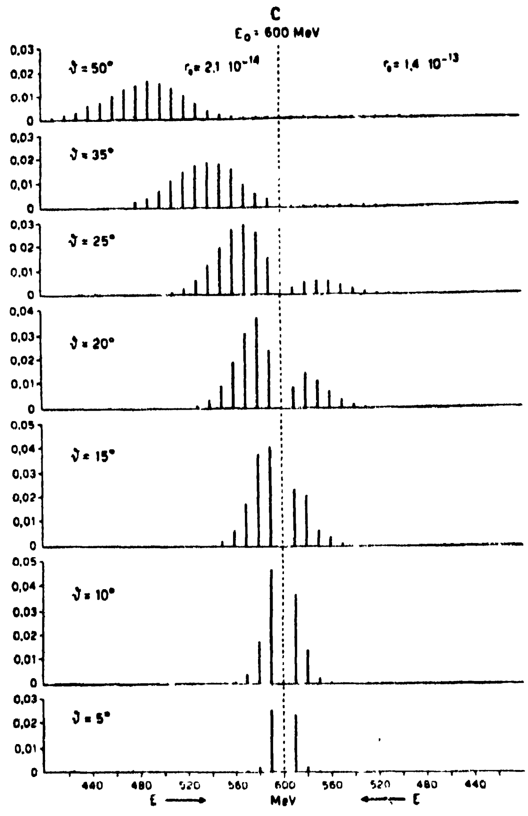

FIG. 2.5

exists for elements up to ^{16}O (*i.e.* nuclei involving only the two first shells) and an almost negligible difference for heavier nuclei.

Then we tried to change the well by introducing a perturbation of the type $V = Ae^{-\beta^2 r^2}$ whose effect is to flatten the bottom of the potential well (figure 2.6) making it a bit more similar to the square well. The calculations have been

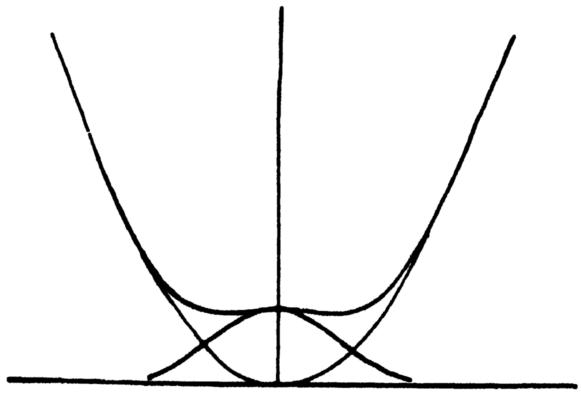

FIG. 2.6

completed for the two first shells. We found that the quantity $\frac{d\sigma}{\sigma} : d\frac{A}{W}$ is rather small ($\sim 4\%$) so that we can conclude that, at least for very light elements, the shape of the potential well is not of primary importance

A third point that is now under consideration is to see what can be the influence of the coupling among different nucleons. The fact that the nucleons have been assumed completely independent of one another could be a too rough picture of the nucleus, especially for the calculation of the inelastic scattering.

(*b*) *The model of the nucleon.* The assumption of a Gaussian distribution of the charge of the proton has only the significance of simplest work assumption, which allows to see the influence of a spatial spread of the electric charge of the proton.

The question has been treated from a more general point of view by Corinaldesi (1951) who has given a purely phenomenological treatment assuming that both the proton and the incident meson have an extended distribution of charge as well as of magnetic moment. Furthermore, in order to preserve the covariance of the formalism, he has assumed that the charge and magnetic moment are spread in time as well as in space.

He has found that, provided the wave function of at least one of the two colliding particles is a plane wave, the relation (2.20) still holds in a generalized form, where four form-factors appear, two for each one of the particles, namely the first representing the charge distribution, the second representing the magnetic moment distribution.

Corinaldesi's result puts the phenomenological theory on sound basis: however its generality spoils in some way the advantages of a phenomenological theory whose main interest has to be looked for in the possibility of determining, by comparison with the results of some convenient scattering experiments, the minimum number of parameters characterising the structure of the considered particles.

A considerable simplification of Corinaldesi's formula can be obtained considering that the coupling between the μ mesons and the fields of the π mesons and the electrons is so weak that in a first approximation we can try to assume the μ as point-charged particles. If such a simplification is allowed, the generalized expression given by Corinaldesi would still contain two form-factors representing the structure of the nucleon : namely its charge distribution and its magnetic moment distribution.

A final remark still in the spirit of the phenomenological representation of the structure of the nucleons, is suggested by the consideration of the neutrons present in the nuclei. If the spread of the charge (and magnetic moment) of a nucleon is mainly due to processes of emission and absorption of mesons and if the coupling constant between the nucleon and the mesonic field is so weak that the only important states are those with 0 or 1 (Frölich, Heitler and Kemmer, 1938) emitted mesons, we can describe the charge density of a nucleon surrounded by its mesonic field as the sum of charge densities (multiplied by convenient weights) of the "pure proton state" and of each one of the different types of emitted mesons.

For instance, if the emitted mesons are of a single type, say π, we have for a proton

$$\rho_p = t\rho_P + (1-t)\rho_\pi$$

and for a neutron

$$\rho_n = (1-t)(\rho_P - \rho_\pi)$$

where ρ_P is the charge distribution of the pure proton state, ρ_π the charge distribution of the emitted π meson, and t the fraction of time during which the proton is in the pure proton state and the neutron in the pure neutron state.

Considering that in the nuclear structure the protons and the neutrons are paired so that the two partners of each pair have almost exactly the same wave function, we must expect interference effects between the waves scattered by the two partners. One can easily recognize that in a case like that considered above, the coherent scattering due to an even-even nucleus is independent, in this approximation, from t and ρ_π and corresponds exactly to the

TABLE 2.2

r_0 as deduced by comparison with the results of mesonic theory

$r_0 \times 10^{14}$ cm

$E = 300$ MeV; $\lambda^* = 6.57 \times 10^{-14}$ cm

ϑ	Scalar charged	Scalar neutral	Pseudoscalar charged	Pseudoscalar neutral
20°	10	3 8	3.8	3.8
40°	7.2	3.3	3.3	2.7
60°	6.9	3.0	3 2	2.7
$E = 600$ MeV; $\lambda^* = 3.28 \times 10^{-14}$ cm				
20°	7 6	3 2	3 8	2 7
40°	6 5	2 8	3 3	2.4
60°	5 4	2.7	2 9	2.3
$E = 940$ MeV; $\lambda^* = 2.1 \times 10^{-14}$ cm				
20°	6 9	3.2	3.2	2.7
40°	5.3	2.7	2 9	2.5
60°	4 2	2 5	2 6	2.1

scattering of Z protons existing during all the time in the "pure proton state".

Similar interference effects, although more complicated, can be expected also for the incoherent scattering.

If the coupling between the nucleons and the mesonic field is so strong that we can no more neglect the states in which 2 or more mesons are emitted, the total charge densities of a nucleon cannot be represented as a linear combination of the densities relative to each one of the corresponding partial states. A detailed discussion about what happens in this case needs a deeper insight into the nature of the mesonic field employed. The only qualitative consideration that we like to add, is that, if the coupling between nucleons and mesonic field is very strong, the exchange of mesons among the nucleons inside the nucleus can be so frequent that the charge distribution tends to spread over the whole volume of the nucleus itself.

2.5—*Comparison of the Phenomenological Theory with some Results obtained by Mesonic Field Theory*

Although the main advantage of the phenomenological point of view used above consists in its capacity to supply expressions of the cross section to be compared with experimental results, which are independent from the uncertainties of the mesonic field theory, it seems desirable to compare the results of the few calculations available today based on the mesonic theory, with some very simple phenomenological assumption, let us say for instance the Gaussian assumption.

The scattering of electrons of a few hundred MeV by protons has been calculated by Rosenbluth (1950); a similar calculation for μ mesons has been performed by Corinaldesi (1951).

The first of these authors gives graphs of the effective proton charge as a function of the energy E and the scattering angle ϑ of the incident electron for both charged and neutral meson theories of scalar and pseudoscalar type, with the coupling constant chosen to fit the magnitude of the observed proton anomalous magnetic moment.

By comparison of these graphs with the Gaussian assumption, one can derive the values of the phenomenological radius r_0 of the proton as a function of E and ϑ. Table 2.2 shows that r_0 does not change very much by changing the angle of observation ϑ and the energy of the impinging electrons. Therefore, considering the great uncertainties implicitly contained in perturbation theory calculations like that of Rosenbluth and Corinaldesi, it seems reasonable, at the moment to use the phenomenological theory with the Gaussian assumption for comparison with experimental results and try to derive from these last the quantity r_0 as a function of E and ϑ. At a later time these results could be compared with the provisions of the various mesonic theories.

The more promising procedure would be, of course, to perform first scattering experiments of fast electrons or μ mesons, by hydrogen in order to get direct information on the electromagnetic structure of the proton and at a second time to repeat experiment of the same type with light nuclei in order to get informations on the nuclear structures and particularly on the interference effects discussed in section 2.4.

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